Mathematical Statistics Qualifier Examination
(Part I of the STAT AREA EXAM)
January 24, 2018; 9:00AM – 11:00AM

Name: ______________________ **ID:** ______________ **Signature:** ________________

Instruction: There are 4 problems – you are required to solve them all. Please show detailed work for full credit. This is a close book exam from 9am to 11am. You need to turn in your exam by 11am, and subsequently, receive the questions for your applied statistics exam. Please do NOT use calculator or cell phone. Good luck!

1. Let $X_1, \cdots, X_n \overset{i.i.d.}{\sim} N(\mu, \sigma^2)$ where both parameters are unknown.
   (a) Please derive whether the sample mean $\bar{X}$ and the sample variance $S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$ would reach the Cramer-Rao lower bound.
   (b) Please derive whether the sample mean and sample variance are UMVUE for (μ, σ^2).

2. For a population with finite fourth moments, please derive the asymptotic distribution of the sample variance $S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$.

3. Please derive:

   (a) The Bayes estimator with respect to the quadratic loss.
   (b) The Bayes estimator with respect to the absolute loss.
   (c) The Bayes estimator with respect to the 0-1 loss.

4. Let $X_1, X_2, \ldots X_n \sim N(\mu, \sigma^2)$, where σ^2 is known
   (a) Find the LRT for $H_0: \mu \leq \mu_0 \quad vs \quad H_a: \mu > \mu_0$
   (b) Show that the test in (a) is a UMP test.

*** That's all, folks! ***