Internal Financing, Managerial Compensation and Multiple Tasks

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SANDRO BRUSCO, FAUSTO PANUNZI

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Stony Brook University

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Sandro Brusco[§] Fausto Panunzi[¶]

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Abstract

We study the optimal capital budgeting policy of a firm taking into account the choice between internal and external financing. The manager can dedicate effort either to increase the short-term profitability of the firm, thus generating greater immediate cash-flow, or to improve long-term perspectives. When both types of effort are observable, low return firms end up using internal funds, while high return firms use external capital markets. When effort to boost short-term cash flow is observable, while effort to boost long-term profitability is not, non-monotonic policies may be optimal, that is. financing switches back and forth between internal and external funds as the quality of the project increases.

1 Introduction

Capital budgeting, the allocation of capital to different projects, is one of the most important activities inside a firm. It is well-known that headquarters and division managers may have different objectives and asymmetric information can lead to an inefficient allocation of capital. The literature on capital budgeting has not paid much attention to the sources of the funds used to finance projects.

In this paper we study the relationship between headquarters and a division. Headquarters is interested in maximizing the value of the firm, net of the compensation paid to the division manager. The division manager derives utility from compensation and from the capital allocated to the division and derives disutility from exerting effort.

One important assumption is that the manager can exert two types of effort: one aimed at improving the quality of new projects and the other at producing

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[§]Department of Economics and College of Business, Stony Brook University.

[¶]Dipartimento di Economia, Università Bocconi, IGIER & CEPR.

cash flow generated by the assets in place. The cash flow can be used to finance new projects. The profitability of the investment depends both on its quality, an exogenous parameter observed only by the divisional manager, and by the level of project-improving effort selected by the manager. Managerial disutility is a convex function of the sum of the two types of effort, so that exerting higher effort in one dimension increases the marginal cost of exerting effort in the other dimension.

We first analyze the case of complete information, that is the quality of the project and the levels of the two types of effort are observed by headquarters. In that case, low quality projects are financed only by internal funds and the associated project-improving effort is also low. The intuition is that low quality projects require low investment. Since the marginal return on project improving effort is positively related to the amount of investment, project improving effort is also low. It is therefore optimal to allocate the division manager effort to generate internal funds. As the quality of the investment projects increases, optimal investment increases. A higher quality of the investment also requires a higher level of project-improving effort. The effect is to increase the marginal cost of the effort aimed at producing cash and thus the firm will rely more on external financing.

We next assume that the effort to generate short-term cash can be easily monitored, while effort directed at project improvement cannot be observed, so that eliciting a higher project-improving effort implies higher incentive costs than eliciting short-term cash generation. With incomplete information an additional effect is at work. Whenever a positive project-improving effort is required, incentive rents have to be paid. As it is well known, it is particularly costly to provide incentive rents for low realizations of the variable on which the agent has private information (the quality of investment project) since this increases the incentive rents for all the higher realizations of the variable. Since incentive rents are related to the total cost of effort, this incentive cost makes it more costly to require both cash–generating and project–improving effort when the quality of the project is low.

Under incomplete information the optimal policy may imply non monotonicity: financing switches back and forth between internal and external funds as the quality of the project increases. At least under some configuration of the parameters, small firms and large firms will mostly rely on external finance. Only firms of intermediate size rely on internal financing to a significant extent.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the general model. Section 4 discusses the optimal policy when information is complete. Section 5 contains the results for the incomplete information case. Conclusions are in section 6. All proofs are collected in the appendix.

2 Literature

The distortions created by asymmetric information on capital budgeting have been studied in a number of papers (see e.g. Harris, Kriebel and Raviv [10], Harris and Raviv [11], and Harris and Raviv [12]). Bernardo, Cai and Luo [4] and Bernardo, Cai and Luo [6] are the papers closest to ours. In particular, Bernardo, Cai and Luo [6] discusses a model which has similar features: the manager has two types of effort, an 'entrepreneurial' effort intended to find investment projects of higher quality and a 'managerial' effort intended to improve the cash flow of a project once it has been selected. A crucial difference is that in their case the two types of effort are taken sequentially and enter separately into the utility function of the manager, thus eliminating most of the trade-offs arising in the presence of multiple tasks. In our case the two types of effort are taken simultaneously and the disutility for the manager depends on total effort. Furthermore, while our project improving effort plays the same role as the 'managerial effort' in Bernardo, Cai and Luo [6], the role of the cash-generating effort is completely different from their 'entrepreneurial' effort. It is precisely the presence of the cash-generating effort that allows us to link the analysis of the multiple tasks problem with the analysis of the choice between internal and external capital markets.

Almazan, Chen and Titman [1] also consider the role of a project-improving effort that interacts with project quality. However they don't have multiple tasks for the manager and they restrict the set of compensation schemes allowed. Begenau and Palazzo [3] analyze the interaction between internal and external financing but, differently from us, they do not consider the managerial effort that goes into the production of internal funds. In their paper the cost of internal funds is that money kept inside the firm has a rate of return inferior to the risk-free rate.

Our paper is also related to the literature on internal capital markets. Various papers (see e.g. Stein [19] and Inderst and Klein [15]) have shown that internal capital markets may be prone to lobbying by divisional managers and thereby subject to a form of redistribution in favor of the divisions with the weakest investment projects. Brusco and Panunzi [8] argue that even if capital is allocated to the divisions with the best investment opportunities, internal capital markets can generate inefficiencies, as the mere possibility of redistribution of funds across division may hinder the effort of divisional managers. In other words, it is the competition among divisions for scarce funds that may hamper the efficiency of the allocation of capital inside firms. This contrasts with our paper where we study only the relationship between headquarters and one division. We preserve however an important feature of Brusco and Panunzi [8], namely that the generation of internal funds requires costly managerial effort, while most of the literature takes the presence of internal funds as given.

Our model is also related to the literature on the pecking order of financial sources. The classical paper by Myers and Majluf [14] shows that, with asymmetric information between firm and investors, it is optimal to use first internal funds, then issue new debt, with equity being the last choice, given its high sensitivity to private information. In our model, in the case of complete information, the optimal mix of internal and external funds depends on the project quality. When quality is low, the investment is also low and the manager must optimally spend little effort on improving it. The manager should rather focus on generating internal funds, as the cost of effort is initially lower than the cost of external funds. As the quality of the project improves, both the investment and the project-improving effort increase and less effort is optimally devoted to generate internal funds. Finally, very high-quality projects require a large investment and are optimally financed only through external funds. While the prediction is the same as the one in Myers and Majluf, we emphasize that the mechanism is completely different. The preference for using internal funds first does not come from asymmetric information but from the fact that at low levels of effort the marginal cost of producing internal funds is low.

3 The Model

The firm has a two-period horizon. In period 1 the firm has assets in place which a risk-neutral manager can use to produce cash flow. The amount of cash flow generated depends on the effort exerted by the manager. Furthermore, it is known that an investment opportunity of stochastic value will appear in the second period. The intrinsic quality of the project is represented by a realization of a random variable $\widetilde{\theta}$ which is private information of the manager. The value of $\widetilde{\theta}$ is drawn from the interval $\left[\underline{\theta}, \overline{\theta}\right]$ according to the density function $f(\theta)$. Let $F(\theta)$ be the cumulative distribution function. We make the following standard assumption.

Assumption 1 *The density $f(\theta)$ is everywhere differentiable, the inverse hazard rate $\mu(\theta) = \frac{1-F(\theta)}{f(\theta)}$ is decreasing in θ and $\mu(\underline{\theta})$ is finite.*

At $t = 1$, the manager can exert two types of effort. The first is a 'cash producing' effort, e_c, leading to an increase in the amount of cash produced by the assets already in place and available for investment or distribution to shareholders at time 2. The cash flow produced is $C = e_c$, so that this type of effort e_c is observable. The second is a 'project improving' effort e_p, leading to an increase in the profitability of the investment project available at time 2. When $\widetilde{\theta}$ takes value θ and the manager exerts project improving effort e_p, the revenue obtained in the second period investing an amount of capital k is

$$V(\theta, e_p, k, \widetilde{u}) = \left(\theta + e_p^{\alpha}\right) k^{\gamma} \widetilde{u} \tag{1}$$

where $\widetilde{u}$ is a noise term distributed on $(0, +\infty)$ and such that $E[\widetilde{u}] = 1$. We will assume that $\gamma \in (0, 1)$ and $\alpha \in (0, 1)$ so that marginal returns to capital and project improving effort are both decreasing. Finally we assume that the two random variables $\widetilde{\theta}$ and $\widetilde{u}$ are independent and the risk-free interest rate is zero. This specification of the function V preserves all the important features of the functional form used in Bernardo, Cai and Luo [4] (henceforth, BCL): As in their model, capital and managerial effort are complementary (the marginal product of one input is increasing in the level of the other input) and the marginal product of capital is increasing in the quality of the project θ.

As in Harris and Raviv [11], Harris and Raviv [12] and BCL [4] we assume that managers derive utility from monetary payments and from controlling larger (higher k) and more profitable (higher θ) projects. More precisely, we assume that the utility for the manager is

$$U(w, e_c, e_p, \theta, k) = w + \zeta\theta k - \frac{1}{2}(e_c + e_p)^2$$

where w is the monetary transfer, k is the capital allocated for investment and $e_c + e_p$ is the total amount of effort. The parameter $\zeta \geq 0$ captures the manager's preference for capital (empire building). Note that we assume that the two types of effort are perfect substitutes from the manager's point of view. The reservation utility is normalized to 0.

Capital can be obtained either through generation of cash flow in the first period (internal financing) or by obtaining funds in the capital markets (external financing). We assume that the firm has to pay an expected rate of return $r \geq 0$ on external funds, which we take as given[1]. If the total amount of capital invested is $k \leq e_c$ then no external financing is necessary and the firm can invest the excess cash $e_c - k$ at the risk-free interest rate.[2] Otherwise, the firm obtains an amount $d = k - e_c$ of external funds on which it pays the expected return r. Thus, we can write the cost of capital in excess of the cost of generating the amount e_c internally as

$$c(k, e_c) = \max\{k - e_c, 0\}(1 + r) - \max\{e_c - k, 0\}.$$

In order to simplify the analysis, we will make the following assumptions on the parameters.

Assumption 2 *The parameters $\gamma, \zeta, \alpha, \underline{\theta}$ and $\overline{\theta}$ satisfy the following inequalities:*

1. $(\gamma + \zeta)\underline{\theta} > 1$
2. $\zeta\overline{\theta} < 1 + r$.
3. $\gamma + \alpha < 1$.

As we will see, inequality (1) implies that, absent incentive problems, the manager is never asked to exert a positive effort e_c unless the resulting cash flow is reinvested in the firm. Inequality (2) makes sure that the optimal amount

[1] We do not model explicitly the expected rate of return r paid by the firm on external capital markets. In our model the risk free rate is zero, so that $r > 0$ implies that the firm is paying a risk premium. However, we allow for the case $r = 0$; the conclusions in that case are essentially the same as in the case $r > 0$.

[2] We do not consider the possibility that the internal funds generated in one division can be allocated to a different division. This case is analyzed in [8]

of capital is finite. Since $\zeta\theta$ is the constant marginal benefit of capital for the manager, if $1+r < \zeta\theta$ then it would be optimal to borrow an infinite amount of capital. Inequality (3) is also needed to make sure that the problem has a finite solution, avoiding increasing returns when the firm increases jointly e_p and k.

For a given investment k, effort e_p and financing policy (e_c, d) the gross expected profit (before paying any managerial compensation) of the firm is

$$\pi(\theta, e_c, e_p, k) = \left(\theta + e_p^{\alpha}\right) k^{\gamma} - c(k, e_c).$$

Headquarters can provide incentives through the compensation contract and the investment and financing policy, designing an investment policy $k\left(\widehat{\theta}\right)$ based on the manager's report about the project quality and a compensation schedule $w\left(V, \widehat{\theta}\right)$ depending on the report and the outcome of the investment. Summing up, the timing of the model is the following:

Period 1. The manager observes the value of θ and headquarters offers to the manager a mechanism

$$\left\{w\left(V, \widehat{\theta}, e_c\right), e_c\left(\widehat{\theta}\right), e_p\left(\widehat{\theta}\right), k\left(\widehat{\theta}\right)\right\}.$$

If the manager does not accept then the game stops. In case of acceptance:

- The manager issues a report $\widehat{\theta}$.
- Headquarters makes an effort recommendation $\left(e_c\left(\widehat{\theta}\right), e_p\left(\widehat{\theta}\right)\right)$.
- The manager chooses effort (e_c, e_p).

Period 2. Headquarters observes e_c. Knowing $\widehat{\theta}$ and e_c headquarters:

- Borrows $d\left(\widehat{\theta}\right)$ and invests $k\left(\widehat{\theta}\right)$, where $d\left(\widehat{\theta}\right) + e_c \geq k\left(\widehat{\theta}\right)$.
- After V is realized, it pays the compensation $w\left(V, \widehat{\theta}, e_c\right)$ to the manager.

In the rest of the paper we will ignore the dependence of w on e_c and we will just assume that the manager always takes the prescribed cash-producing effort. Since e_c is verifiable, this is easily obtained by setting a large punishment for the manager if e_c is not met.

A crucial difference with BCL [4] and BCL [6] is that in our framework the headquarters may ask the manager to generate internally the funds for investment through a costly effort. This provides a way to restrain the tendency for the manager to overstate the quality of the project in order to obtain a higher level of capital. We will first analyze the complete information problem, that is the optimal policy obtained when θ, e_p and e_c are observable. Next we will look at the asymmetric information case, in which only e_c and V are observable.

4 Optimal Policy under Complete Information

The first best solution maximizes, for each θ, the sum of the expected value of the firm V and private benefits to the manager $\zeta\theta k$, net of the cost of effort:

$$\max_{e_c, e_p, k} \quad \left(\theta + e_p^{\alpha}\right) k^{\gamma} + \zeta\theta k - c\left(k, e_c\right) - \frac{1}{2}\left(e_c + e_p\right)^2 \tag{2}$$

s.t.

$$k \geq 0, \;\; e_c \geq 0, \; e_p \geq 0.$$

In order to characterize the first best, we first prove that it is never optimal to choose $e_c > k$.

Lemma 1 *Let $\left(e_c^*, e_p^*, k^*\right)$ be a solution to problem (2). Then $e_c^* \leq k^*$.*

The intuition for the lemma is that the assumption $(\gamma + \zeta)\,\underline{\theta} > 1$ guarantees that the marginal productivity of capital is sufficiently high at low levels of capital, thus implying that at any solution capital is sufficiently large (more precisely, $k > 1$). On the other hand, if $e_c > k$ then the marginal cost of effort at the optimum must be 1, and this in turn implies that e_c, and therefore k, must be small (i.e. $k < e_c \leq 1$). But k cannot be too small, since in that case its marginal productivity is higher than the marginal cost of effort.

Lemma 1 implies that the headquarters will never ask the manager to produce funds which are not reinvested in the firm.[3] Thus, the amount of investment is always given by the full amount e_c of internally generated funds plus additional funds raised in the capital markets, if any. Thus, problem (2) can be more conveniently written as

$$\max_{e_c, e_p, d} \quad \left(\theta + e_p^{\alpha}\right)\left(e_c + d\right)^{\gamma} + \zeta\theta\left(e_c + d\right) - d\left(1 + r\right) - \frac{1}{2}\left(e_c + e_p\right)^2 \tag{3}$$

s.t.

$$e_c \geq 0, \;\; e_p \geq 0, \; d \geq 0.$$

In order to examine the optimal policy under complete information, it is useful to consider first the special case in which $e_p = 0$, i.e. no project–improving effort is allowed. Under this assumption, the problem boils down to choosing the optimal investment size and the financing mix.

When $e_p = 0$, the marginal benefit of increasing k is given by $\gamma\theta k^{\gamma-1} + \zeta\theta$. The marginal cost of obtaining an additional unit of capital is e_c whenever internal funds are used and $1+r$ whenever external funds are used. Headquarters will optimally choose the cheapest source of funds, which means that as long as $k < 1 + r$ the firm will use internal funds and after that it will use external funds. Thus, the marginal cost of capital is $\min\left\{k, 1 + r\right\}$. Equating marginal revenue and marginal cost we have

$$\frac{\gamma\theta}{k^{1-\gamma}} + \zeta\theta = \min\left\{k, 1 + r\right\}. \tag{4}$$

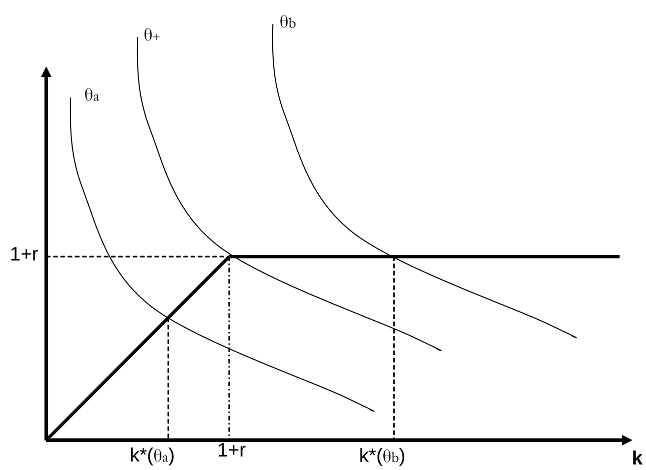
[3] This result would not hold in case headquarters can reallocate funds across divisions.

Call $k^*(\theta)$ the unique implicit solution to this equation. Notice that, since the left hand side is strictly increasing in θ while the right hand side does not depend on θ, the function $k^*(\theta)$ is strictly increasing in θ. The next proposition describes the optimal policy for this case.

Proposition 1 *Suppose $e_p = 0$. Let $k^*(\theta)$ be the solution to equation (4) and define θ^+ as the value such that $k^*(\theta^+) = 1 + r$. The optimal policy can be described as follows:*

1. *If $\theta < \theta^+$ then $e_c^*(\theta) = k^*(\theta) < 1 + r$ and $d^*(\theta) = 0$.*
2. *If $\theta \geq \theta^+$ then $e_c^*(\theta) = 1 + r$, $d^*(\theta) = k^*(\theta) - (1 + r) > 0$.*

The solution is quite intuitive. When $e_p = 0$ then the marginal cost of obtaining internally generated funds is e_c, while the marginal cost of obtaining external funds is $1+r$. Thus the firm uses internal funds up to $1+r$, and external funds afterwards. Whether or not the firm will use external funds depends on the productivity of capital θ.



The point at which the marginal cost crosses the marginal revenue gives the optimal amount of capital $k(\theta)$. For low values of θ the productivity is low; in this case the optimal amount of capital is less than $1+r$ and only internal funds are used (this is the case labeled θ_a in the picture). Higher values of θ shift outward the marginal benefit curve. Thus, the optimal investment increases and the firm uses both internal and external funds; specifically, internal cash production is pushed up to the point where the marginal cost of internal cash generation is equal to the constant marginal cost of external funds.

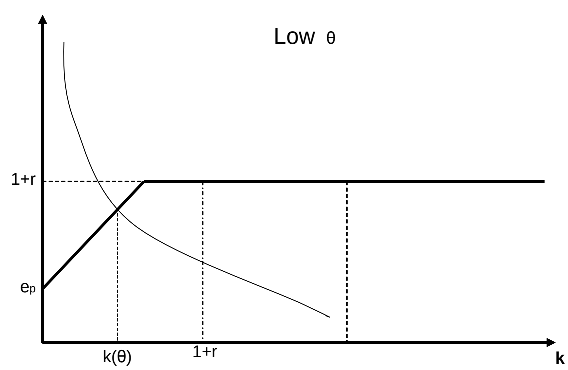
We now move to the case where the division manager can also improve the quality of the project by exerting an effort e_p. When e_p is increased, both the marginal cost and the marginal benefit of capital are affected. The marginal cost of internal fund generation increases, becoming $e_c + e_p$, while the cost of external funds remains constant at $1+r$. This means that increasing e_p shifts financing from internal to external capital. If we only consider the cost effect, an increase in e_p in principle can decrease the optimal investment k. However, an increase in e_p also raises the marginal productivity of capital. This tends to increase the optimal investment size.

Given our functional specification, the optimal amount of e_p must be strictly positive whenever $k > 0$, since the marginal benefit of e_p (given by $\alpha e_p^{\alpha-1} k^{\gamma}$) goes to infinity as e_p goes to zero. Furthermore, investment and project improving effort are complements: a higher level of k increases the marginal benefit of e_p, and a higher level of e_p increases the marginal benefit of k. This means that, compared to the situation in which e_p is zero, investment will typically be higher and financing will more frequently come from external sources. The optimal policy is described in the following proposition.

Proposition 2 *The first best policy $e_p(\theta), e_c(\theta)$ and $k(\theta)$ is as follows. The functions $e_p(\theta)$ and $k(\theta)$ are increasing. There are two threshold values θ^a and θ^b such that:*

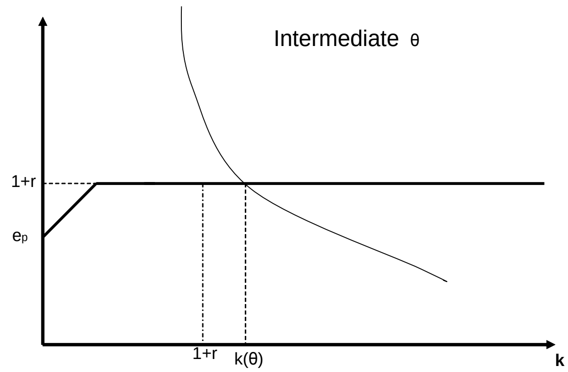
- *If $\theta < \theta^a$ then $e_c(\theta) = k(\theta)$ and $d(\theta) = 0$. Total effort $e_p(\theta) + e_c(\theta)$ is increasing in θ and strictly less than $1+r$.*
- *If $\theta \in \left[\theta^a, \theta^b\right]$ then the cash producing effort is given by $e_c(\theta) = 1 + r - e_p(\theta)$ and debt is given by $d(\theta) = \left(\frac{1+r}{\alpha}\right)^{\frac{1}{\gamma}} \left[e_p(\theta)\right]^{\frac{1-\alpha}{\gamma}} - e_c(\theta)$.*
- *If $\theta > \theta^b$ then the cash producing effort is $e_c(\theta) = 0$, debt is $d(\theta) = k(\theta)$ and $e_p(\theta) > 1 + r$.*

Under the optimal policy the interval $\left[\underline{\theta}, \overline{\theta}\right]$ is partitioned in three sub-intervals. In the first subinterval, $[\underline{\theta}, \theta^a]$, the value of θ is low and the optimal amount of investment is low.



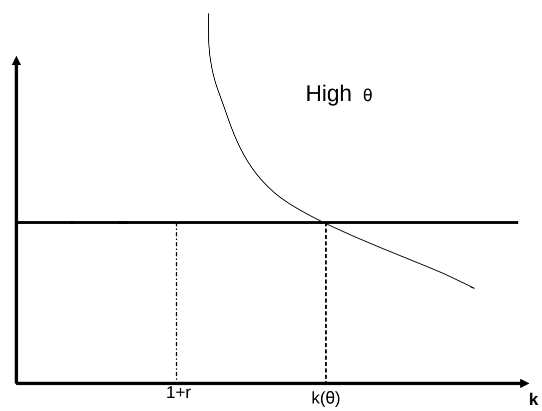
When the optimal investment is relatively low, the amount of project improving effort is also low. This makes sure that the cost of internal funds remains low.

Thus, the firm finances entirely the investment with internal capital $e_c(\theta)$. As θ increases we move to the second interval $\left(\theta^a, \theta^b\right)$.



In this interval the optimal investment is high enough to require external funding. Both internal and external funding are used and the total amount of effort is $1+r$. For higher values of θ the amount of investment increases and this, given the complementarity between investment and project improving effort, induces higher levels of e_p. Since the total amount of effort is constant, internal funding decreases in θ.

The third interval $\left[\theta^b, \overline{\theta}\right]$ correspond to the set of values in which capital is very productive.



In this case investment is very high and therefore the optimal e_p is also high. In fact, it is higher than $1+r$, which makes the marginal cost of external financing always smaller than the marginal cost of internal financing. Thus, when θ is very high the financing comes entirely from external sources and managerial effort is devoted exclusively to improving the profitability of the firm.

After having characterized the optimal capital budgenting and financing policies under complete information, we turn to the study of asymmetric information.

5 Optimal Policies under Incomplete Information

We now analyze the optimal mechanism for the case in which headquarters does not observe the project quality θ and the effort e_p, while effort e_c is verifiable. The manager reports about project quality, and we denote by $\widehat{\theta}$ the announcement. Since the cash-producing effort is verifiable, we can assume that, whenever $\widehat{\theta}$ is reported, the manager is forced to take effort $e_c\left(\widehat{\theta}\right)$; for example, the contract may specify a large fine[4] if the manager does not produce an amount of cash $e_c\left(\widehat{\theta}\right)$. The capital allocation policy is a function $k\left(\widehat{\theta}\right)$ determining the amount of capital given to the manager as a function of the announcement $\widehat{\theta}$. Let V be a realization of the revenue function $V\left(\theta, e_p, k, \widetilde{u}\right)$ defined by (1). The compensation scheme is a function $w\left(V, \widehat{\theta}\right)$. We will make the following assumption.

Assumption 3 *The manager can destroy, without being observed, part of the revenue. Thus, only compensation schemes $w\left(V, \widehat{\theta}\right)$ which are weakly increasing in V can be incentive compatible.*

This assumption is common in the literature, see e.g. Innes [13]. It is realistic if the manager can manipulate the actual or observed revenue of the firm through cost overrun, window dressing and so on. Notice however that we assume that revenue cannot be directly stolen by the manager.

Since the manager is risk neutral, the only thing that matters is the expected value of the salary. Let

$$w^e\left(\theta, e_p, \widehat{\theta}\right) = \int_0^{+\infty} w\left(\left(\theta + e_p^\alpha\right) k^\gamma\left(\widehat{\theta}\right) u, \widehat{\theta}\right) f(u)\, du$$

be the expected salary when the true state is θ, project improving effort e_p is undertaken and the manager has reported $\widehat{\theta}$. Using the change of variable

[4] Given our assumption of risk neutrality it would be very easy to accomodate the case in which the principal, instead of observing e_c, is only able to observe a noisy signal of the cash producing effort, say $\widetilde{s} = e_c + \widetilde{\varepsilon}$, where $\widetilde{\varepsilon}$ is a random variable with finite variance σ_ε^2 and mean 0, independent of $\widetilde{\theta}$ and $\widetilde{u}$. The principal can make managerial compensation depend on the realization s of $\widetilde{s}$. As an example of how to induce a desired level of $\overline{e}_c$ at basically no cost, suppose that a quadratic function

$$h(s) = A - B\left(s - \overline{e}_c\right)^2$$

is added to the compensation schedule, where $B > 0$ is large and $A = B\sigma_\varepsilon^2$. When the agent chooses the prescribed level of effort $\overline{e}_c$ then $E\left[h\left(\widetilde{s}\right)\right] = 0$. If the agent chooses $\widehat{e}_c \neq \overline{e}_c$ then

$$E\left[h\left(\widetilde{s}\right)\right] = -B\left(\widehat{e}_c - \overline{e}_c\right)^2$$

and for large values of B the cost of deviating from $\overline{e}_c$ becomes very high. While the actual compensation scheme will be more complicated, the main point is that our analysis will go through as long some signal of e_c, independent of θ and e_p, can be oserved. Of course this will no longer be true if the agent is risk-averse.

$v = \left(\theta + e_p^\alpha\right) k^\gamma \left(\widehat{\theta}\right) u$, the expected salary can be written as

$$w^e\left(\theta, e_p, \widehat{\theta}\right) = \int_0^{+\infty} w\left(v, \widehat{\theta}\right) \frac{f\left(\frac{v}{\left(\theta+e_p^\alpha\right)k^\gamma\left(\widehat{\theta}\right)}\right)}{\left(\theta + e_p^\alpha\right) k^\gamma \left(\widehat{\theta}\right)} dv. \tag{5}$$

Notice that w^e may be smooth with respect to θ and e_p even if the function w is not (the only requirement on w is that the integral defined in (5) exist). The next lemma establishes that this is in fact the case.

Lemma 2 *For each $\widehat{\theta}$ the function $w^e\left(\cdot, \cdot, \widehat{\theta}\right)$ is differentiable with respect to θ and e_p and*

$$\frac{\partial w^e}{\partial e_p} = \frac{\partial w^e}{\partial \theta} \alpha e_p^{\alpha-1}$$

whenever $e_p > 0$. Furthermore, for every incentive-compatible mechanism we have

$$\left.\frac{\partial w^e\left(\theta, e_p, \widehat{\theta}\right)}{\partial \theta}\right|_{\left(e_p, \widehat{\theta}\right) = \left(e_p(\theta), \theta\right)} = \frac{1}{\alpha}\left(e_c(\theta) + e_p(\theta)\right) e_p^{1-\alpha}(\theta).$$

The lemma provides the 'envelope condition' that can be used to determine the rate of growth of expected utility in any truth-telling mechanism. Let

$$U^{**}\left(\theta, \widehat{\theta}, e_p\right) \equiv w^e\left(\theta, e_p, \widehat{\theta}\right) + \zeta\theta k\left(\widehat{\theta}\right) - \frac{\left(e_c\left(\widehat{\theta}\right) + e_p\right)^2}{2}$$

denote the expected utility of the manager when she observes θ, reports $\widehat{\theta}$ and takes effort pair $\left(e_c\left(\widehat{\theta}\right), e_p\right)$. An effort function $e_p(\theta)$ is implementable if we can find two functions k and w such that individual rationality and incentive compatibility are satisfied. Individual rationality requires

$$U^{**}\left(\theta, \theta, e_p(\theta)\right) \geq 0 \qquad \forall \theta \in \left[\underline{\theta}, \overline{\theta}\right],$$

while incentive compatibility requires

$$\left(\theta, e_p(\theta)\right) \in \arg \max_{\widehat{\theta}, e_p} \; U^{**}\left(\theta, \widehat{\theta}, e_p\right) \qquad \forall \theta \in \left[\underline{\theta}, \overline{\theta}\right].$$

Define the optimal choice of effort for the manager $e_p\left(\theta, \widehat{\theta}\right)$ as

$$e_p\left(\theta, \widehat{\theta}\right) \in \arg\max_{e_p} \; U^{**}\left(\theta, \widehat{\theta}, e_p\right),$$

that is $e_p\left(\theta, \widehat{\theta}\right)$ is the optimal effort of a manager who has observed θ and announced $\widehat{\theta}$. Now define

$$U^*\left(\theta, \widehat{\theta}\right) \equiv U^{**}\left(\theta, \widehat{\theta}, e_p\left(\theta, \widehat{\theta}\right)\right) \qquad \text{and} \qquad U(\theta) \equiv U^*(\theta, \theta).$$

Lemma 2 has the following important implication.

Proposition 3 *The function $U(\theta)$ is differentiable and convex. The derivative is*

$$U'(\theta) = \frac{e_c(\theta) + e_p(\theta)}{\alpha} e_p^{1-\alpha}(\theta) + \zeta k(\theta). \tag{6}$$

The proposition gives a road-map for computing the optimal policy. Write the problem as

$$\max_{w(\cdot),k(\cdot),e_c(\cdot),e_p(\cdot)} E\left[\left(\theta + e_p^{\alpha}(\theta)\right) k^{\gamma}(\theta) - c\left(e_c(\theta), k(\theta)\right) - w^e\left(\theta, e_p(\theta), \theta\right)\right]$$

subject to:

$$\theta \in \arg\max_{\widehat{\theta}} \quad U^*\left(\theta, \widehat{\theta}\right) \qquad \forall \theta \in \left[\underline{\theta}, \overline{\theta}\right] \tag{7}$$

$$U(\theta) \geq 0 \qquad \forall \theta \in \left[\underline{\theta}, \overline{\theta}\right] \tag{8}$$

$$k(\theta) \geq 0, \qquad e_c(\theta) \geq 0, \qquad e_p(\theta) \geq 0 \qquad \forall \theta \in \left[\underline{\theta}, \overline{\theta}\right] \tag{9}$$

Let

$$w^e(\theta) = w^e\left(\theta, e_p(\theta), \theta\right).$$

Since

$$U(\theta) = w^e(\theta) + \zeta\theta k(\theta) - \frac{\left(e_c(\theta) + e_p(\theta)\right)^2}{2},$$

we can write

$$w^e(\theta) = U(\theta) - \zeta\theta k(\theta) + \frac{\left(e_c(\theta) + e_p(\theta)\right)^2}{2}.$$

Integrating by parts we have

$$E\left[U(\theta)\right] = \int_{\underline{\theta}}^{\overline{\theta}} U(\theta) f(\theta) d\theta = \int_{\underline{\theta}}^{\overline{\theta}} U'(\theta)\left(1 - F(\theta)\right) d\theta.$$

Using Proposition 3 the problem for the headquarters can be written as

$$\max_{k(\cdot),e_c(\cdot),e_p(\cdot)} E\left[\left(\theta + e_p^{\alpha}(\theta)\right) k^{\gamma}(\theta) + \zeta\theta k(\theta) - c\left(e_c(\theta), k(\theta)\right) - \frac{\left(e_c(\theta) + e_p(\theta)\right)^2}{2}\right]$$
$$-E\left[\left(\frac{e_c(\theta) + e_p(\theta)}{\alpha} e_p^{1-\alpha}(\theta) + \zeta k(\theta)\right)\mu(\theta)\right] \tag{10}$$

subject to $U(\underline{\theta}) \geq 0$ and $U'(\theta)$ non-decreasing, where $\mu(\theta) = \frac{1-F(\theta)}{f(\theta)}$ is a decreasing function of θ.

The objective function in the optimization problem (10) is written to emphasize the incentive costs. The part inside the first expectation is exactly what the problem would look like under complete information. The additional component on the second row is $E\left[U'(\theta)\mu(\theta)\right]$ and it represents the additional cost that comes from the existence of incomplete information. The intuition is that a type $\theta' > \theta$ can always pretend to be type θ, so if we increase the attractiveness of reporting θ for a type θ' then we have to increase the utility of type θ' in order to maintain the incentives to truth-telling. As a consequence, whenever the allocation gives an extra additional utility to a type θ then it has to give the same additional utility to all types above θ, a mass $1 - F(\theta)$. Thus, the incentive cost of changing the allocation in a way that increases by $U'(\theta)$ the utility of type θ is $U'(\theta)(1 - F(\theta))$. The inverse hazard rate $\frac{1-F(\theta)}{f(\theta)}$ comes from the fact that we want to write the expectation using the distribution $f(\theta)$.

5.1 The Structure of Optimal Mechanisms

The structure of the optimal mechanism may vary depending on the parameters. We will first establish some general facts and next we will present an example.

Let us consider the problem ignoring the constraint that $U'(\theta)$ should be increasing. When we do that the problem can be solved pointwise, so we have

$$\max_{e_c, e_p, k} \quad \left(\theta + e_p^{\alpha}\right) k^{\gamma} + \zeta\theta k - c\left(e_c, k\right) - \left(\zeta k + \frac{1}{\alpha}\left(e_c + e_p\right) e_p^{1-\alpha}\right) \mu\left(\theta\right) - \frac{\left(e_c + e_p\right)^2}{2} \tag{11}$$

subject to

$$e_c \geq 0, \quad e_p \geq 0, \quad k \geq 0$$

for each value of θ. Since the marginal return on capital goes to $+\infty$ as k tends to 0 and $\mu(\theta)$ is finite, any solution must have $k > 0$. Thus, the contraint $k \geq 0$ can be ignored. The Lagrangian associated to the problem is

$$\begin{aligned} L &= \left(\theta + e_p^{\alpha}\right) k^{\gamma} + \zeta\left(\theta - \mu\left(\theta\right)\right) k - c\left(e_c, k\right) - \frac{e_c + e_p}{\alpha} e_p^{1-\alpha} \mu\left(\theta\right) \\ &\quad - \frac{\left(e_c + e_p\right)^2}{2} + \lambda_c e_c + \lambda_p e_p. \end{aligned}$$

Notice that the function $c(e_c, k)$ is not differentiable at $e_c = k$ whenever $r > 0$.

Remark. Differently from the complete information case, $e_p = 0$ may be part of an optimal solution. For this to happen it has to be the case that $\lim_{e_p \downarrow 0} \frac{\partial L}{\partial e_p} \leq 0$ when L is evaluated at the optimal policy. This is in principle possible because, while the marginal return of a small increase in e_p (given by $\alpha e_p^{\alpha-1} k^{\gamma}$) goes to infinity as $e_p \downarrow 0$ when $k > 0$, the marginal incentive cost (given by $\frac{1-\alpha}{\alpha}\mu(\theta) e_c e_p^{-\alpha}$) also goes to infinity when $e_c > 0$, which must be the case when $e_p = 0$. On the other hand, if

$$\lim_{e_p \downarrow 0} \quad \left(\alpha e_p^{\alpha-1} k^{\gamma} - \frac{1}{\alpha}\mu\left(\theta\right) e_p^{1-\alpha} - \frac{1-\alpha}{\alpha}\left(e_c + e_p\right)\mu\left(\theta\right) e_p^{-\alpha} - \left(e_c + e_p\right)\right) > 0.$$

for each $k > 0$ and $e_c \geq 0$ then the optimal policy must involve $e_p > 0$. By inspection, we observe that for each $k > 0$ the condition is satisfied whenever $e_c = 0$. This is an intuitive result; if effort is not spent generating cash it must be spent improving the project, since when $e_c = e_p = 0$ the marginal cost of effort is zero. When $e_c > 0$ the condition is equivalent to

$$\lim_{e_p \downarrow 0} \quad e_p^{-\alpha} \left(\alpha e_p^{2\alpha-1} k^{\gamma} - \left(\frac{1-\alpha}{\alpha} e_c \mu(\theta) \right) \right) > 0.$$

If $\alpha < \frac{1}{2}$ the condition is always satisfied, while for $\alpha > \frac{1}{2}$ it is never satisfied. The case $\alpha = \frac{1}{2}$ is knife-edge and it depends on the sign of $\left(\frac{k^{\gamma}}{2} - \mu(\theta) e_c\right)$.

We start stating a result for the cases in which the optimal policy requires only external financing.

Proposition 4 *Suppose that on an interval* $\left(\theta^a, \theta^b\right)$ *the optimal policy is such that* $e_c(\theta) = 0$. *Then the optimal policy is given by a strictly increasing function* $\overline{e}_p(\theta)$ *such that*

$$\overline{e}_p(\theta) + \frac{1}{\alpha} \mu(\theta) \left(\overline{e}_p(\theta)\right)^{1-\alpha} \geq 1 + r \tag{12}$$

for each $\theta \in \left(\theta^a, \theta^b\right)$ *and by a strictly increasing capital function* $\overline{k}(\theta)$ *given by*

$$\overline{k}(\theta) = \left(\frac{\gamma\left(\theta + \left(\overline{e}_p(\theta)\right)^{\alpha}\right)}{1 + r - \zeta(\theta - \mu(\theta))} \right)^{\frac{1}{1-\gamma}}. \tag{13}$$

When the firm is using only external financing the marginal cost of capital is constant. A higher θ both increases the marginal product of capital and it decreases the agency cost of the project-improving effort, given that $\mu(\theta)$ is decreasing. Since k and e_p are complementary they must be both increasing with θ. The function $\overline{e}_p(\theta)$ is implicitly defined as the unique solution to an equation resulting from the first order conditions (see equation (47) in the appendix).

While in general we cannot say that the optimal financing policy will start with internal financing at low values of θ and end up with external financing at high levels of θ, we can provide some conditions for this to be true. These are collected in the next Proposition.

Proposition 5 *Suppose*

$$\overline{e}_p(\underline{\theta}) + \frac{1}{\alpha} \mu(\underline{\theta}) \left(\overline{e}_p(\underline{\theta})\right)^{1-\alpha} < 1 + r. \tag{14}$$

Then there is an interval $[\underline{\theta}, \theta^a)$ *such that internal financing is used. Furthermore, suppose that under complete information the optimal policy prescribes external financing only. Then there is an interval* $\left(\theta^b, \overline{\theta}\right]$ *such that the optimal policy under complete information prescribes external financing only.*

When inequality (14) holds then $e_c = 0$ cannot be part of the optimal policy at $\underline{\theta}$, as established by Proposition 4. Thus, internal financing must be used at low values of θ. Furthermore, if only external financing is used for high values of θ when information is complete then the same must be true under incomplete information for sufficiently high values of θ, since $\mu(\theta)$ goes to zero and the difference between the complete information problem and the incomplete information problem vanishes.

When inequality (14) does not hold then it is possible to have an optimal policy using external financing only at low levels of θ. In general, notice that condition (12) requires either a high value of e_p or a low value of θ (implying a high value of $\mu(\theta)$). This points out the possibility of non-monotonic policies, i.e. policy in which the firm uses exclusively external financing when θ is low or high but it uses both internal and external financing for intermediate values of θ. We provide below an example that illustrates this situation.

The intuition for the non-monotonicity of the use of external funds with respect to project quality is the following. When θ is low, the full information level of project-improving effort is low. But, on the other hand, $\mu(\theta)$ is high so that a high incentive rent must be paid to the division manager. In order to reduce the rent, it may be optimal to use only external funds, without generating cash flow internally, as this reduce the marginal cost of the project-improving effort. As θ increases, the full information level of project-improving effort increases, but $\mu(\theta)$ decreases and thus the agency problem becomes less severe. Then the use of internal funds may become optimal, as they may be initially cheaper than external ones. Finally, for very high values of θ, the full information value of project-improving effort becomes high and, to curb the disutility of effort of the division manager, it may be optimal to rely only on external funds.

5.2 Non-monotonicity. An Example.

Assume $\zeta = 0$ and $r = 0$, so that the cost of capital function becomes

$$c(k, e_c) = k - e_c$$

and the objective function is everywhere differentiable. Furthermore set $\gamma = \frac{1}{2}$ (so that $k^{1-\gamma} = k^{\gamma}$) and $\alpha = \frac{1}{3}$. Notice that, by the previous remark, this implies that the optimal $e_p(\theta)$ is always strictly positive, so at an optimum we must have $\frac{\partial L}{\partial e_p} = 0$.

The distribution of θ has support on the interval $[3, +\infty)$ and the density function is given by

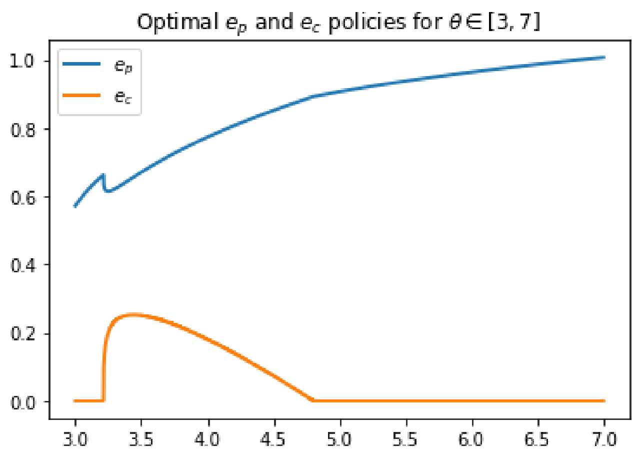
$$f(\theta) = c(4\theta - 11)\, e^{11\theta - 2\theta^2}$$

where c is the normalization constant $c = e^{-11\times 3 + 2\times 3^2}$. The cumulative distribution function and the inverse hazard rate function are given by

$$F(\theta) = 1 - ce^{11\theta - 2\theta^2} \qquad \mu(\theta) = \frac{1}{4\theta - 11}.$$

In this case the optimal policy can be computed numerically. The results for the interval $[3, 7]$ are given in the following picture[5].

[5] On the interval $(7, +\infty)$ the optimal policy has $e_c = 0$ and e_p increasing.



The optimal policy always has $e_p > 0$, since $\alpha < \frac{1}{2}$. This implies that the marginal cost of cash production is always strictly positive. The optimal policy has three intervals. On the first interval $\mu(\theta)$ is high and this leads to the choice of $e_c = 0$, since incentive costs are too high. However in the example $\mu(\theta)$ decreases very quickly, so it reaches low values when θ is still relatively low. At low levels of θ the optimal capital allocation $k(\theta)$ is relatively low and the optimal level of $e_p(\theta)$ is also relatively low. This leaves room for a strictly positive level e_c. Finally, as θ increases both $k(\theta)$ and $e_p(\theta)$ increase, making the marginal cost of effort high. As $\mu(\theta)$ fades, the optimal policy converges to the one under complete information.

Remark. Notice that in this example the optimal policy $e_p(\theta)$ is **not** monotonic. Proposition 4 states that $e_p(\theta)$ must be increasing when $e_c = 0$ but it does not say that $e_p(\theta)$ should be increasing globally. In this example $e_p(\theta)$ decreases around the point at which $e_c(\theta)$ becomes strictly positive, a consequence of the fact that the marginal cost of effort strongly increases.

5.3 Implementing the Second Best Policy

Let $e_c(\theta)$, $e_p(\theta)$ and $k(\theta)$ be the optimal second best policy under incomplete information. Let

$$\phi(\theta) = \frac{e_c(\theta) + e_p(\theta)}{\alpha} e_p^{1-\alpha}(\theta) + \zeta k(\theta),$$

which must be a non-decreasing function. We have the following result.

Proposition 6 *The optimal policy under incomplete information can be implemented by an affine compensation function, with coefficients depending on the announcement $\widehat{\theta}$.*

The compensation function $w\left(V,\widehat{\theta}\right)$ yielding the optimal policy can be written as

$$w\left(V,\widehat{\theta}\right)=a\left(\widehat{\theta}\right)+d\left(\widehat{\theta}\right)V$$

where

$$d\left(\widehat{\theta}\right)=\frac{\phi\left(\widehat{\theta}\right)-\zeta k\left(\widehat{\theta}\right)}{\left(k\left(\widehat{\theta}\right)\right)^{\gamma}}$$

and

$$\begin{aligned} a\left(\widehat{\theta}\right) &= \frac{1}{2}\left(e_c\left(\widehat{\theta}\right)+e_p\left(\widehat{\theta}\right)\right)^2+\zeta e_p^{\alpha}\left(\widehat{\theta}\right)k\left(\widehat{\theta}\right)-d\left(\widehat{\theta}\right)\left(\widehat{\theta}+e_p^{\alpha}\left(\widehat{\theta}\right)\right)k^{\gamma}\left(\widehat{\theta}\right) \\ &\quad +\int_{\underline{\theta}}^{\widehat{\theta}}\phi\left(s\right)ds \end{aligned}$$

The function $a\left(\theta\right)$ is the fixed wage and it is chosen so that the manager is left with exactly the incentive rents implied by the derivative in (6). Proposition 6 implies that Headquarters does not have to concede more than that amount to the manager.

The sensitivity of managerial wage to θ, given by $d\left(\theta\right)$, need not be monotonic. Even the expected total compensation linked to results $E\left[d\left(\theta\right)V\right]$ need not be monotonic. When $\widehat{\theta}=\theta$ we have

$$\begin{aligned} E\left[d\left(\theta\right)V\right] &= \left(\phi\left(\theta\right)-\zeta k\left(\theta\right)\right)\left(\theta+e_p^{\alpha}\left(\theta\right)\right) \\ &= \left(\frac{e_c\left(\theta\right)+e_p\left(\theta\right)}{\alpha}e_p^{1-\alpha}\left(\theta\right)\right)\left(\theta+e_p^{\alpha}\left(\theta\right)\right). \end{aligned}$$

The value of $E\left[d\left(\theta\right)V\right]$ may decrease in θ at points in which an increase in θ requires a decrease in $e_p\left(\theta\right)$, i.e. at points at which the optimal policy requires the manager to put more effort in cash production and less in project improvement.

6 Conclusion

This paper studies the oprimal capital budgeting policy of a firm where the choice between internal and external financing is explicitly modeled. A division manager can exercise different types of effort, aimed either at immediate results and therefore to ready-to-use funds, or to improve the long-run prospects of the firm. This second type of effort is much more difficult to observe than the first one and therefore requires the payment of incentive rents.

We first characterize the optimal policy when both types of effort are fully observable. In this case what we observe is that firms which already have a high expected return on investment ask the manager to work to improve the quality of the project rather than to generate cash and therefore rely more on external financing rather than on internal financing. The reason is that a high expected return implies a higher investment, and a higher investment increases the marginal return of the effort put in improving the long run profitability of the firm. Thus, in the case in which all types of effort are observable, firms with low *ex ante* returns have a low capital investment, a low level of effort dedicated to improving the productivity of capital and a low level of external financing, as managerial effort is dedicated mostly to generate fund internally. The opposite occurs with firms with high *ex ante* returns.

Things become more complicated when the different types of effort have different levels of observability. In particular, when effort to generate cash is observable but effort to improve project quality is not, it is necessary to pay incentive rents to the manager in order to increase the type of non-observable effort. In this case the optimal policy may not be monotonic, meaning that the use of internal funds is not monotonically related to the ex ante profitability of investment. We provide an example that illustrates this possibility. When the firm has low expected return it will be relatively small in size and it will have a very high incentive cost of effort. The firm thus prefers to save on the marginal cost of effort by not generating internal funds and focusing managerial effort on improving productivity. When the expected return is very high we also have zero cash production, as high effort aimed at improving the quality of the project is optimally required. Internal financing may instead occur at intermediate levels of productivity, when the relatively low level of capital implies that project-improving effort is not very productive but incentive costs are not very high so that the incentive cost of cash production is low.

Appendix I

Proof of Lemma 1. First notice that, in general, we can ignore the positivity constraint $k \geq 0$ since the marginal utility of capital at $k = 0$ tends to $+\infty$. If the solution is $e_c^* > k^*$ then the positivity constraint on e_c can also be ignored. Thus, if at the solution we have $e_c^* > k^*$ then the solution should be obtained solving the maximization problem:

$$\max_{e_p, e_c, k} \quad \left(\theta + e_p^{\alpha}\right) k^{\gamma} + \zeta\theta k + e_c - k - \frac{1}{2}\left(e_c + e_p\right)^2$$

$$\text{s.t.} \qquad e_p \geq 0.$$

The first order conditions are

$$\frac{\gamma\left(\theta + e_p^{\alpha}\right)}{k^{1-\gamma}} + \zeta\theta = 1 \tag{15}$$

$$e_c + e_p = 1 \tag{16}$$

$$\alpha e_p^{\alpha-1} k^{\gamma} + \lambda = e_c + e_p \tag{17}$$

$$\lambda \geq 0, \qquad e_p \geq 0, \qquad \lambda e_p = 0. \tag{18}$$

If the solution is $e_c^* > k^*$, then equation (16) implies $k^* < 1$. However, the right-hand side of equation (15) is decreasing in k, and at $k = 1$ we have

$$\gamma\left(\theta + e_p^{\alpha}\right) + \zeta\theta > 1.$$

The inequality follows from Assumption 2-1 and $e_p \geq 0$. We conclude that $e_c^* > k^*$ implies $k^* > 1$, a contradiction. ■

Proof of Proposition 1. The objective function is concave and the constraint set is convex, so the first order conditions are necessary and sufficient for an optimum. If $e_p = 0$ then the Lagrangian of the problem is

$$L = \theta\left(e_c + d\right)^{\gamma} + \zeta\theta\left(e_c + d\right) - d\left(1 + r\right) - \frac{1}{2}e_c^2 + \lambda_c e_c + \lambda_d d$$

and the first order conditions are

$$\gamma\theta\left(e_c + d\right)^{\gamma-1} + \zeta\theta + \lambda_c = e_c \tag{19}$$

$$\gamma\theta\left(e_c + d\right)^{\gamma-1} + \zeta\theta + \lambda_d = 1 + r \tag{20}$$

$$\lambda_c \geq 0, \quad \lambda_d \geq 0, \quad e_c \geq 0, \quad d \geq 0, \quad \lambda_c e_c = 0, \quad \lambda_d d = 0. \tag{21}$$

From (19) it follows that $e_c > 0$, so that $\lambda_c = 0$. Furthermore, if $\lambda_c = 0$ then subtracting (19) from (20) we obtain

$$\lambda_d = 1 + r - e_c, \tag{22}$$

which implies $e_c \leq 1+r$. For each θ, let $k^*(\theta)$ be the solution to equation (4). There are two possibilities. First, we might have

$$\gamma\theta(1+r)^{\gamma-1} + \zeta\theta < 1+r. \tag{23}$$

In that case the total amount of capital $e_c + d$ must be strictly less than $1+r$; if not, equation (20) requires $\lambda_d > 0$ and $d = 0$, but then equation (19) can't be satisfied. Furthermore, it must be $d = 0$. If not, we would have $\lambda_d = 0$ and $e_c < 1+r$, so that the two first order conditions would be incompatible. We conclude that in this case the solution is $e_c^*(\theta) = k^*(\theta)$ and $d^*(\theta) = 0$.

The other case is when inequality (23) does not hold. In this case equation (19) can only be satisfied if $e_c + d \geq 1+r$ and $\lambda_d = 0$. From (22) we have $e_c = 1+r$, and both (19) and (20) become equivalent to (4). Thus in this case the solution is $e_c^*(\theta) = 1+r$ and $d^*(\theta) = k^*(\theta) - (1+r)$. Since θ^+ is defined by the condition

$$\gamma\theta^+(1+r)^{\gamma-1} + \zeta\theta^+ = 1+r$$

then clearly the solution will be $e_c^*(\theta) = k^*(\theta)$, $d^*(\theta) = 0$ when $\theta < \theta^+$ and $e_c^*(\theta) = 1+r$, $d^*(\theta) = k^*(\theta) - (1+r)$ when $\theta \geq \theta^+$. Otherwise, we set $\theta^+ = \overline{\theta}$ when (23) is satisfied for each θ and $\theta^+ = \underline{\theta}$ if (23) is never satisfied. ■

Proof of Proposition 2. As in the previous case, the first order conditions are necessary and sufficient for an optimum. The Lagrangian is

$$L = \left(\theta + e_p^\alpha\right)(e_c + d)^\gamma + \zeta\theta(e_c + d) - d(1+r) - \frac{1}{2}(e_c + e_p)^2 + \lambda_c e_c + \lambda_d d + \lambda_p e_p$$

and the first order conditions are

$$\gamma\left(\theta + e_p^\alpha\right)(e_c + d)^{\gamma-1} + \zeta\theta + \lambda_c = e_c + e_p \tag{24}$$

$$\alpha e_p^{\alpha-1}(e_c + d)^\gamma + \lambda_p = e_c + e_p \tag{25}$$

$$\gamma\left(\theta + e_p^\alpha\right)(e_c + d)^{\gamma-1} + \zeta\theta + \lambda_d = 1+r. \tag{26}$$

From (24) we conclude $e_c + d > 0$, and this in turn implies from (25) that $e_p > 0$ and $\lambda_p = 0$.

Consider first the case in which the positivity constraints do not bind, so that $\lambda_c = \lambda_d = 0$. Equations (24) and (26) imply

$$e_c + e_p = 1+r.$$

In turn this can be substituted in (25) to obtain

$$e_c + d = \left(\frac{1+r}{\alpha}\right)^{\frac{1}{\gamma}} e_p^{\frac{1-\alpha}{\gamma}},$$

which can be substituted into (26) to get the equation

$$\theta e_p^{\frac{(1-\alpha)(\gamma-1)}{\gamma}} + e_p^{\frac{\alpha+\gamma-1}{\gamma}} = \frac{(1+r-\zeta\theta)(1+r)^{\frac{1-\gamma}{\gamma}}}{\gamma\alpha^{\frac{1-\gamma}{\gamma}}}. \tag{27}$$

Since $\alpha + \gamma < 1$, the left hand side is strictly decreasing in e_p and it goes from $+\infty$ to 0 as e_p moves from 0 to $+\infty$; the right hand side is positive because of Assumption 2–2. Therefore, for each θ there is a unique solution, which we call $e_p^*(\theta)$. The function $e_p^*(\theta)$ is increasing in θ, since the left hand side is increasing in θ and the right hand side is decreasing.

This solution is feasible if

$$e_c^*(\theta) = 1 + r - e_p^*(\theta) \geq 0 \tag{28}$$

and

$$d^*(\theta) = \left(\frac{1+r}{\alpha}\right)^{\frac{1}{\gamma}} \left[e_p^*(\theta)\right]^{\frac{1-\alpha}{\gamma}} + e_p^*(\theta) - (1+r) \geq 0 \tag{29}$$

Equation (28) requires $e_p^*(\theta) < 1 + r$, while equation (29) requires $e_p^*(\theta)$ to be sufficiently high (at $e_p = 1 + r$ inequality (29) is satisfied, so the range of values of e_p that satisfies the two inequalities is non-empty).

If we now define

$$\theta^a = \inf\left\{\theta \left| \left(\frac{1+r}{\alpha}\right)^{\frac{1}{\gamma}} \left[e_p^*(\theta)\right]^{\frac{1-\alpha}{\gamma}} + e_p^*(\theta) \geq (1+r) \right.\right\} \tag{30}$$

and

$$\theta^b = \sup\left\{\theta \left| 1 + r \geq e_p^*(\theta) \right.\right\}, \tag{31}$$

then we can conclude that for each value $\theta \in \left[\theta^a, \theta^b\right]$ the solution is given by the unique global unconstrained optimum of the objective function. More precisely, when $\theta \in \left[\theta^a, \theta^b\right]$ the solution is to set e_p equal to $e_p^*(\theta)$ (the solution to equation (27)), $e_c^*(\theta)$ as given by (28) and $d^*(\theta)$ as given by (29).

Next assume that the positivity constraints bind, i.e. $\theta < \theta^a$ or $\theta > \theta^b$. Then, exactly one of the two positivity constraints will be binding. Consider first the case $e_c = 0$, $d > 0$. For this case the first order conditions are

$$\gamma\left(\theta + e_p^\alpha\right) d^{\gamma-1} + \zeta\theta + \lambda_c = e_p \tag{32}$$

$$\alpha e_p^{\alpha-1} d^\gamma = e_p \tag{33}$$

$$\gamma\left(\theta + e_p^\alpha\right) d^{\gamma-1} + \zeta\theta = 1 + r. \tag{34}$$

Since $\lambda_c \geq 0$, equations (32) and (34) imply $e_p \geq 1 + r$. From (33) we obtain

$$d = \alpha^{-\frac{1}{\gamma}} e_p^{\frac{2-\alpha}{\gamma}},$$

and substituting back into (34) we obtain

$$\theta e_p^{\frac{(2-\alpha)(\gamma-1)}{\gamma}} + e_p^{\frac{\alpha+2\gamma-2}{\gamma}} = \frac{1 + r - \zeta\theta}{\gamma\alpha^{\frac{1-\gamma}{\gamma}}}. \tag{35}$$

Equation (35) has a unique solution, call it $\overline{e}_p(\theta)$, which is strictly increasing in θ. Remember that the solution is feasible only if $e_p \geq 1 + r$, and observe that, given the definition of θ^b, we have $\overline{e}_p\left(\theta^b\right) = 1 + r = e^*\left(\theta^b\right)$. This implies that the solution is feasible whenever $\theta \geq \theta^b$ and it is not feasible when $\theta < \theta^b$. In fact, if $\theta > \theta^b$ we have $e_p(\theta) > 1 + r$.

Consider next the case $e_c > 0$, $d = 0$. In this case the first order conditions are

$$\gamma\left(\theta + e_p^{\alpha}\right)e_c^{\gamma-1} + \zeta\theta = e_c + e_p \tag{36}$$

$$\alpha e_p^{\alpha-1}e_c^{\gamma} = e_c + e_p \tag{37}$$

$$\gamma\left(\theta + e_p^{\alpha}\right)e_c^{\gamma-1} + \zeta\theta + \lambda_d = 1 + r. \tag{38}$$

Since $\lambda_d \geq 0$, equations (36) and (38) imply $e_c + e_p \leq 1 + r$. Let $E = e_c + e_p$ be total effort. Then, from (37) we have

$$e_c = \left(\frac{E}{\alpha}\right)^{\frac{1}{\gamma}} e_p^{\frac{1-\alpha}{\gamma}} \tag{39}$$

and substituting in (36) we obtain

$$\theta e_p^{\frac{(1-\alpha)(\gamma-1)}{\gamma}} + e_p^{\frac{\alpha+\gamma-1}{\gamma}} = \frac{(E-\zeta\theta)\,E^{\frac{1-\gamma}{\gamma}}}{\gamma\alpha^{\frac{1-\gamma}{\gamma}}}. \tag{40}$$

For each θ and $E > \zeta\theta$ there is a unique value $e_p(\theta, E)$ solving (40). Plugging this solution into (39) we obtain a unique value for e_c, call it $e_c(\theta, E)$. Define the function

$$\Phi(\theta, E) = e_p(\theta, E) + e_c(\theta, E).$$

The function Φ is continuous in E and defined over the interval $(\zeta\theta, +\infty)$. Furthermore, for each E the function Φ is strictly increasing in θ. This is because at a fixed level of E the value $e_p(\theta, E)$ that solves (40) increases in θ and $e_c(\theta, E)$ depends on θ only through e_p.

Since feasibility requires $E \leq 1 + r$, a feasible solution exists if the equation

$$E = \Phi(\theta, E) \tag{41}$$

has a solution $E^* \leq 1 + r$. The right hand side goes to $+\infty$ as E goes to $\zeta\theta$. If at $E = 1 + r$ we have $\Phi(\theta, 1 + r) > 1 + r$ then no solution exists. This is because in this case we would have at least two feasible solutions; this would be equivalent to having two optima for a strictly concave function, which is impossible. Thus, a necessary condition for a feasible solution to exist is that $\Phi(\theta, 1 + r) \leq 1 + r$. Given the continuity of Φ the condition is also sufficient. Now observe that if a feasible solution to (41) exists for a given value of θ' then it must exists for all values $\theta < \theta'$. This is because the function Φ is increasing in θ, so that $\Phi\left(\theta', 1 + r\right) \leq 1 + r$ implies $\Phi(\theta, 1 + r) < 1 + r$ for each $\theta < \theta'$. At last, observe that at the value θ^a defined by (30) we have that $E(\theta^a) = 1 + r$ is a solution to (41). This implies that for $\theta > \theta^a$ any solution to (41) must involve $E(\theta) > 1+r$, and it is therefore not feasible. On the other hand, if $\theta < \theta^a$ then a feasible solution exists. ■

Proof of Lemma 2. Since we assumed that the density f is differentiable, using the expression of $w^e\left(\theta, e_p, \widehat{\theta}\right)$ given in (5) it is immediate to see that $w^e\left(\theta, e_p, \widehat{\theta}\right)$ is differentiable with respect to the first two arguments and that

$$\frac{\partial w^e}{\partial e_p} = \frac{\partial w^e}{\partial \theta}\alpha e_p^{\alpha-1} \tag{42}$$

for each $e_p > 0$.

Suppose now that at θ the project improving effort is $e_p(\theta) > 0$. A necessary condition for implementability is that $e_p(\theta)$ maximizes the expected utility of the manager when the true θ is reported, i.e.

$$e_p(\theta) \in \arg\max_{e_p} \quad w^e(\theta, e_p, \theta) + \zeta\theta k(\theta) - \frac{(e_c(\theta) + e_p)^2}{2}$$

Since w^e is differentiable, a necessary condition for optimality is

$$\frac{\partial w^e(\theta, e_p, \theta)}{\partial \theta}\alpha e_p^{\alpha-1} = e_c(\theta) + e_p$$

where we have made use of (42). This implies

$$\frac{\partial w^e(\theta, e_p, \theta)}{\partial \theta} = \frac{e_c(\theta) + e_p(\theta)}{\alpha} e_p^{1-\alpha}(\theta)$$

whenever $e_p(\theta) > 0$.

If $e_p(\theta) = 0$ then a necessary condition for $e_p = 0$ to be optimal is that

$$\frac{\partial w^e(\theta, e_p, \theta)}{\partial \theta}\alpha e_p^{\alpha-1} \leq e_c(\theta) + e_p \tag{43}$$

for each $e_p > 0$. Since $e_p^{\alpha-1}$ goes to $+\infty$ as e_p goes to zero and Assumption 3 implies that $\frac{\partial w^e(\theta, e_p, \theta)}{\partial \theta} \geq 0$, it follows that the only way in which (43) can be satisfied is by setting $\frac{\partial w^e(\theta, e_p, \theta)}{\partial \theta} = 0$. Summing up we have

$$\frac{\partial w^e(\theta, e_p, \theta)}{\partial \theta} = \frac{e_c(\theta) + e_p(\theta)}{\alpha} e_p^{1-\alpha}(\theta).$$

for each θ. ■

Proof of Proposition 3. Let

$$U^{**}\left(\theta, \widehat{\theta}, e_p\right) = w^e\left(\theta, e_p, \widehat{\theta}\right) + \zeta\theta k\left(\widehat{\theta}\right) - \frac{\left(e_c\left(\widehat{\theta}\right) + e_p\right)^2}{2}$$

By Lemma 2 the function $w^e\left(\theta, e_p, \widehat{\theta}\right)$ is differentiable with respect to θ. It follows that $U^{**}\left(\theta, \widehat{\theta}, e_p\right)$ is differentiable with respect to θ. The envelope theorem then implies

$$U'(\theta) = \left.\frac{\partial U^{**}\left(\theta, \widehat{\theta}, e_p\right)}{\partial \theta}\right|_{\left(e_p, \widehat{\theta}\right)=\left(e_p(\theta), \theta\right)} = \left.\frac{\partial w^e\left(\theta, e_p, \widehat{\theta}\right)}{\partial \theta}\right|_{\left(e_p, \widehat{\theta}\right)=\left(e_p(\theta), \theta\right)} + \zeta k(\theta).$$

We now can use the expression for $\left.\frac{\partial w^e\left(\theta, e_p, \widehat{\theta}\right)}{\partial \theta}\right|_{\left(e_p, \widehat{\theta}\right)=\left(e_p(\theta), \theta\right)}$ given in the statement of Lemma 2 and conclude

$$U'(\theta) = \frac{e_c(\theta) + e_p(\theta)}{\alpha} e_p^{1-\alpha}(\theta) + \zeta k(\theta)$$

at each θ. Convexity follows from standard arguments. ■

Proof of Proposition 4. When $e_c = 0$ the objective function is everywhere differentiable. Suppose that the optimal policy is such that $e_c(\theta) = 0$ on the interval $\left(\theta^a, \theta^b\right)$. Then, on such interval, the optimal pair $(e_p(\theta), k(\theta))$ must solve

$$\max_{e_p, k} \quad \left(\theta + e_p^\alpha\right) k^\gamma + \zeta \theta k - (1+r) k - \left(\zeta k + \frac{1}{\alpha} e_p^{2-\alpha}\right) \mu(\theta) - \frac{e_p^2}{2}$$

The objective function is supermodular in (e_p, k) and it satisfies increasing differences in $(e_p, k; \theta)$, as it can be easily checked looking at the mixed second derivatives. It follows that the solution is non-decreasing in θ.

Further information on the function $\overline{e}_p(\theta)$ is obtained observing that when $e_c = 0$ the objective function is strictly concave in (k, e_p). The optimal point is therefore given by the unique solution to the first order conditions.

$$\gamma\left(\theta + e_p^\alpha\right) k^{\gamma-1} + \zeta(\theta - \mu(\theta)) = 1 + r \tag{44}$$

$$\alpha e_p^{\alpha-1} k^\gamma = \frac{2-\alpha}{\alpha} \mu(\theta) e_p^{1-\alpha} + e_p \tag{45}$$

From (44) we have

$$k = \left(\frac{\gamma\left(\theta + e_p^\alpha\right)}{1 + r - \zeta(\theta - \mu(\theta))}\right)^{\frac{1}{1-\gamma}} \tag{46}$$

and substituting into (45) and manipulating we obtain

$$\alpha\left(\frac{\gamma\left(\theta + e_p^\alpha\right)}{1 + r - \zeta(\theta - \mu(\theta))}\right)^{\frac{\gamma}{1-\gamma}} = \frac{2-\alpha}{\alpha} \mu(\theta) e_p^{2-2\alpha} + e_p^{2-\alpha} \tag{47}$$

When $\alpha + \gamma \leq 1$ the LHS of (47) is strictly concave and strictly positive at $e_p = 0$, while the RHS is strictly convex and equal to zero at $e_p = 0$. Thus the equation has a unique solution $\overline{e}_p(\theta)$ which is strictly increasing in θ. Inserting this expression into (46) we obtain the solution $\overline{k}(\theta)$.

Finally, notice that for $e_c(\theta) = 0$ to be optimal the first order condition w.r.t. e_c requires

$$\lambda_c = e_p + \frac{1}{\alpha}\mu(\theta) e_p^{1-\alpha} - (1+r) \geq 0, \tag{48}$$

so the function $\overline{e}_p(\theta)$ must satisfy

$$\overline{e}_p(\theta) + \frac{1}{\alpha}\mu(\theta)(\overline{e}_p(\theta))^{1-\alpha} \geq (1+r). \tag{49}$$

Notice that the expression on the LHS of (49) may not be increasing, since $\mu(\theta)$ is decreasing. ■

Proof of Proposition 5. If inequality (14) is satisfied then the first order condition wrt to e_c cannot be satisfied at $e_c = 0$ when $\theta = \underline{\theta}$. Thus, the optimal policy requires at least some internal financing at $\underline{\theta}$. Given the continuity of the objective function, the strict inequality implies that some internal financing must be optimal for value of θ sufficiently close to $\underline{\theta}$.

The second part of the Proposition is a simple application of the 'no distortion at the top' principle. Since $\mu(\overline{\theta}) = 0$ the optimal policy at $\overline{\theta}$ under incomplete information is the same as the optimal policy under complete information. Thus, if it is strictly optimal to adopt external financing only, the continuity of the objective function implies that it is optimal to adopt external financing only for θ sufficiently close to $\overline{\theta}$. ■

Proof of Proposition 6. Let

$$\phi(\theta) = \frac{e_c(\theta) + e_p(\theta)}{\alpha} e_p^{1-\alpha}(\theta) + \zeta k(\theta)$$

Consider a linear compensation rule of the form

$$w\left(V, \widehat{\theta}\right) = a\left(\widehat{\theta}\right) - \zeta\left(k\left(\widehat{\theta}\right)\right)^{1-\gamma} V + d\left(\widehat{\theta}\right) V$$

where

$$d\left(\widehat{\theta}\right) = \phi\left(\widehat{\theta}\right)\left(k\left(\widehat{\theta}\right)\right)^{-\gamma}$$

$$\begin{aligned} a\left(\widehat{\theta}\right) &= \frac{1}{2}\left(e_c\left(\widehat{\theta}\right) + e_p\left(\widehat{\theta}\right)\right)^2 + \zeta e_p^{\alpha}\left(\widehat{\theta}\right) k\left(\widehat{\theta}\right) - d\left(\widehat{\theta}\right)\left(\widehat{\theta} + e_p^{\alpha}\left(\widehat{\theta}\right)\right) k^{\gamma}\left(\widehat{\theta}\right) \\ &\quad + \int_{\underline{\theta}}^{\widehat{\theta}} \phi(s)\, ds \end{aligned}$$

Notice that

$$E\left[w\left(\widehat{\theta},V\right)\right]+\zeta\theta k\left(\widehat{\theta}\right)=a\left(\widehat{\theta}\right)-\zeta e_p^{\alpha}k\left(\widehat{\theta}\right)+d\left(\widehat{\theta}\right)\left(\theta+e_p^{\alpha}\right)k^{\gamma}\left(\widehat{\theta}\right)$$

The expected utility of an agent who observes θ, reports $\widehat{\theta}$ and takes effort $\left(e_c\left(\widehat{\theta}\right),e_p\right)$ can be written as:

$$U\left(\theta,\widehat{\theta},e_p\right)=a\left(\widehat{\theta}\right)-\zeta e_p^{\alpha}k\left(\widehat{\theta}\right)+d\left(\widehat{\theta}\right)\left(\theta+e_p^{\alpha}\right)k^{\gamma}\left(\widehat{\theta}\right)-\frac{\left(e_c\left(\widehat{\theta}\right)+e_p\right)^2}{2}.$$

Thus, after announcing $\widehat{\theta}$ the manager solves

$$\max_{e_p}\quad a\left(\widehat{\theta}\right)-\zeta e_p^{\alpha}k\left(\widehat{\theta}\right)+d\left(\widehat{\theta}\right)\left(\theta+e_p^{\alpha}\right)k^{\gamma}\left(\widehat{\theta}\right)-\frac{\left(e_c\left(\widehat{\theta}\right)+e_p\right)^2}{2}.$$

$$\text{subject to}\qquad e_p\geq 0.$$

If $\widehat{\theta}$ is such that $e_p\left(\widehat{\theta}\right)=0$ then $d\left(\widehat{\theta}\right)=\zeta k^{1-\gamma}\left(\widehat{\theta}\right)$ and the optimal choice is $e_p=0$ for each θ.

If $\widehat{\theta}$ is such that $e_p\left(\widehat{\theta}\right)>0$ then the first order condition for maximization is

$$\left(d\left(\widehat{\theta}\right)k^{\gamma}\left(\widehat{\theta}\right)-\zeta k\left(\widehat{\theta}\right)\right)\alpha e_p^{\alpha-1}=e_c\left(\widehat{\theta}\right)+e_p. \tag{50}$$

Since

$$d\left(\widehat{\theta}\right)k^{\gamma}\left(\widehat{\theta}\right)-\zeta k\left(\widehat{\theta}\right)=\frac{e_c\left(\widehat{\theta}\right)+e_p\left(\widehat{\theta}\right)}{\alpha}e_p^{1-\alpha}\left(\widehat{\theta}\right)>0$$

the equation has a unique solution. In fact, using the definition of $d\left(\widehat{\theta}\right)$ it can be checked that $e_p\left(\widehat{\theta}\right)$ is a solution of (50), so we conclude

$$e_p=e_p\left(\widehat{\theta}\right).$$

Thus, once the manager has announced $\widehat{\theta}$ she will take the prescribed action $e_p\left(\widehat{\theta}\right)$ no matter what the true θ is. Using this fact and the definition of $a\left(\widehat{\theta}\right)$, we can write

$$U\left(\theta,\widehat{\theta}\right)=\int_{\underline{\theta}}^{\widehat{\theta}}\phi\left(s\right)ds+\left(\theta-\widehat{\theta}\right)\phi\left(\widehat{\theta}\right).$$

If $\widehat{\theta}<\theta$ then

$$U\left(\theta,\widehat{\theta}\right)-U\left(\theta\right)=-\int_{\widehat{\theta}}^{\theta}\left(\phi\left(s\right)-\phi\left(\widehat{\theta}\right)\right)ds\leq 0$$

since ϕ is non-decreasing. If $\widehat{\theta}>\theta$ then we have

$$U\left(\theta,\widehat{\theta}\right)-U\left(\theta\right)=\int_{\theta}^{\widehat{\theta}}\left(\phi\left(s\right)-\phi\left(\widehat{\theta}\right)\right)ds\leq 0$$

since ϕ is non-decreasing. Thus, announcing the truth is optimal. ■

Appendix II

In this appendix we describe how the optimal policy described in subsection 5.2 is computed.

When $r = 0$ the objective function is everywhere differentiable and for each pair (θ, e_p) the level of k that maximizes the value of the objective function is given by the first order condition

$$k^{\frac{1}{2}} = \frac{1}{2}\left(\theta + e_p^{\frac{1}{3}}\right)$$

where we used $\gamma = \frac{1}{2}$, $\alpha = \frac{1}{3}$. Using this fact, the objective function can be written as a function of e_p and e_c only:

$$W(e_p, e_c, \theta) = \frac{1}{4}\left(\theta + e_p^{\frac{1}{3}}\right)^2 + e_c - 3(e_c + e_p)\, e_p^{\frac{2}{3}}\mu(\theta) - \frac{(e_c + e_p)^2}{2}$$

Let $x = e_p^{\frac{1}{3}}$ and $y = e_c$. Then the objective function becomes

$$\widehat{W}(x, y, \theta) = \frac{1}{4}(\theta + x)^2 + y - 3\left(y + x^3\right)x^2\mu(\theta) - \frac{\left(y + x^3\right)^2}{2}$$

Now notice that the function is strictly concave in y for each (x, θ) and that the optimal value of y for each given pair (x, θ) is

$$\widehat{y}(x, \theta) = \max\left\{1 - 3x^2\mu(\theta) - x^3, 0\right\}.$$

At this point we can look at the objective function

$$W^*(x, \theta) = \frac{1}{4}(\theta + x)^2 + \widehat{y}(x, \theta) - 3\left(\widehat{y}(x, \theta) + x^3\right)x^2\mu(\theta) - \frac{\left(\widehat{y}(x, \theta) + x^3\right)^2}{2}$$

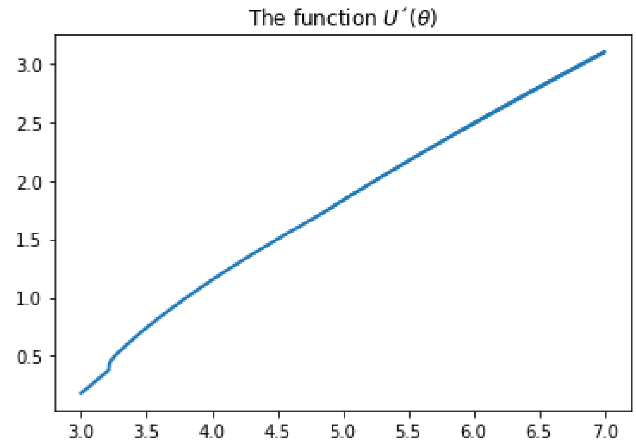
and compute numerically the optimal value $x^*(\theta)$ for each θ. The optimal policy is then given by

$$\begin{aligned} e_p^*(\theta) &= (x^*(\theta))^3 \\ e_c^*(\theta) &= \widehat{y}(x^*(\theta), \theta) \\ k^*(\theta) &= \frac{1}{2}(\theta + x^*(\theta)) \end{aligned}$$

It can also be checked numerically that the expression

$$U'(\theta) = 3\left(e_p^*(\theta) + e_c^*(\theta)\right)\left(e_p^*(\theta)\right)^{\frac{2}{3}}$$

is in fact increasing in θ, as it is shown in the following picture.



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